Evaluating Number Sense in Community College Developmental Math Students

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NumberWorks

Abstract

Community college developmental math students (N = 657) from three math levels were asked to place five whole numbers on a line that had only endpoints 0 and 20 marked. How the students placed the numbers revealed the same three stages of behavior that Steffe and Cobb (1988) documented in determining young children's number sense. 23% of the students showed a lack of the concept of part-whole coexistence in this task. In two of three levels, lack of the concept was found to be significantly related to success (final grade of A, B, or C) in developmental math.

In her review of The Centre for Literacy's 2014 Summer Institute and its focus on data from PIAAC (Program for the International Assessment of Adult Competencies), Tighe (2014) commented that "research is needed ... to adequately design interventions to identify, target, and improve key component numeracy skills" (p. 66) among adult students. In the spirit of that comment, this article describes a practitioner-devised tool, and its use in a community college-sponsored research project to uncover which students appear to lack key numeracy components critical for understanding proportions, fractions and algebraic relationships. Stigler, Givvin, and Thompson (2009) reported a lack of conceptual understanding in those particular areas of pre-college-level math among community college developmental-level math students.

The purpose of this study was to identify how many students in developmental math classes may be lacking key developmental math concepts that standardized skills tests may fail to identify. These concepts are: 1) the "equal distance of 1" that exists between neighboring whole numbers, which is necessary for understanding abstract addition; and 2) part-whole coexistence (the parts and whole exist at the same time), which is necessary for understanding abstract subtraction, fractions and quantities in relationship (percents, ratios, functions, and more).

Over the years, through one-on-one interviews, the author had identified individuals who lacked one or both concepts 1) among High School Equivalency program (GED) math classes, 2) among a sample (N=11) of community college students, 3) among pre-service teachers, and 4) among prisoners transitioning back to society (Steinke, 1999; 2002; 2008). In all these populations, some individuals struggled to answer, or could not answer, the question 7 + ? = 25 that was presented with physical objects and numerals, but not in written form.

Research Question

With the development of the much quicker Number Line Assessment tool, it became practical to attempt identification of concept-lacking adults with a much larger group. The research question was posed as: How many developmental math students lack one or both concepts at the start of the course, and what is the success rate of these students in developmental math classes?

The key purpose is identifying whole number concepts, rather than skills, that adults lack. To understand what these concepts are at the earliest level, we turn to research on young children's number sense carried out in the 1980s by Dr. Leslie Steffe and his colleagues at the University of Georgia.

Conceptual Framework

Mature number sense with whole numbers has been thought to appear around age 7 or 8 (Piaget, 1953). More recent research on brain development pushes that toward age 9 (Houdé et al., 2011), particularly for children who grow up in conditions of toxic stress (poverty and/or abuse) (Child Welfare Information Gateway, 2015). Other recent research relates children's math achievement to non-verbal number sense (Halberda, Mazzocco, & Feigenson, 2008) and to placement of whole numbers on an empty number line (Booth & Siegler, 2008; Mundy & Gilmore, 2009; Rouder & Geary, 2014; Schneider, Grabner & Paetsch, 2009).

In the 1980s, Steffe and his colleagues developed a model of primary-grade children's growth toward number sense (Steffe et al., 1983; Steffe, Thompson & Richards, 1982; Steffe, Richards & von Glasersfeld, 1978). Wright used a variation of Steffe's original model to assess larger groups of children (Wright, 1994). The outcome of those assessments was used to develop a math curriculum in Australia (Wright, 2003). *Math Recovery*, a "Response to Intervention" (RTI) program for the early grades in the United States, is a further extension of Steffe's early model (Miller, 2014; Wright, 2009).

Steffe with Cobb (Steffe & Cobb, 1988) later refined the original model to three stages: perceptual (concrete), figurative (representational) and abstract thinkers. This update was based on behaviors observed in one-on-one interviews in which children answered simple addition or missing addend questions. The concepts that allow students to progress from one stage of number sense to the next are: equal-sized units from one whole number to the next (the concrete-to-figurative transition); and part-whole coexistence (the figurative-to-abstract transition).

In their 3 Stages model, Steffe and Cobb defined children as Stage 1 (perceptual) when the children had acquired the number word sequence and could use it to count with one-to-one correspondence. The researchers documented these counting behaviors with Stage 1 primary grade students: 1) fingers are raised in a "block" for number patterns (i.e., all fingers go up at once); 2) objects must be seen in order to be counted (i.e., objects not in sight are not included in the count); and 3) counting to add starts from 1 each time (i.e., a "count all" strategy).

Older children and adults who understand each counting number as a separate item exhibit Stage 1 behaviors in one-on-one interviews (Steinke, 1999;

Steinke, 2001). These people understand numbers as labels of items in a certain order, like house numbers. For them, there is no exact quantitative distance from number to number. Number words belong to a category, like the names of fruit belong to a category.

Stage 2 also shows specific counting behaviors according to Steffe and Cobb (1988). The person: 1) raises fingers in sequence one after the other when counting; 2) can add unseen objects; 3) "counts on from" one of the addends when adding; 4) substitutes fingers, mental visualizations, or spoken words for unseen objects being counted; and 5) can add parts to find the whole without using physical objects. These behaviors, especially the ability to add unseen objects and "counting on from," would indicate that Stage 2 children and adults have the sense that each counting number is the "same-sized 1 more" than the number before it. That is, since the increase from one number to the next is constant, it doesn't matter where you begin counting when adding two groups of like items. Figure 1 contrasts the physical sense of number relationships between Stage 1 and Stage 2.

Stage 2 thinkers have the first major concept, "equal distance," but lack the second, "part-whole coexistence." Stage 3 thinkers have that second concept, namely, the understanding that a number exists as a whole and at the same time contains within it all the combinations of addends (the parts) that can be summed to create that whole. For example, 11 contains within it 4 + 7 or 3 + 3 + 3 + 2 and many other combinations while existing at the same time as the whole 11.

The important point here is the Stage 3 understanding that the parts and whole exist at the same time as opposed to the Stage 2 understanding that either the parts exist or the whole exists (Fig. 2). Steffe and Cobb (1988) also noted that Stage 3 children could give the solution to a missing addend (subtraction) question on the first try without using counters, and were confident that the answer was correct.

It is the grasp or lack of these two transition concepts ("equal distance" and "part-whole coexistence") that the 5-digit number line assessment reveals. Other researchers have reported tasks with placement of a single number between two designated endpoints in order to show a relationship between students' number sense and their physical placement of numbers relative to each other in space (De Hevia & Spelke, 2009; Longo & Lourenco, 2010). Using 5 digits uncovers much more, and in far less time than interviews.

Method

At a suburban community college, students taking developmental-level math courses (Basic math [whole numbers, fractions and decimals] [*N* = *179*]; Pre-Algebra [*N* = *167*]; Algebra 1 [*N* = *311*]) were assessed for their sense of whole number relationships using an empty number line with endpoints zero and twenty. The college's Institutional Review Board approved the study. Preliminary investigation with four developmental math classes of two different instructors had shown that not all students could place five given whole numbers on the empty line with reasonable accuracy.

The overall student population in the college is about 19% Hispanic and about 2% Black. In the classes that formed the assessment group, the amount of Hispanics was markedly above that 19%: 31.4% of students in Basic Math; 32.9% in Pre-algebra; and 23.3% in Algebra 1. Furthermore, the zip codes of 260 students in eleven Basic Math classes over a period of five years indicate that 33.5% lived in ZIP codes that are in the top 10% of Hispanic percentages of population nationally (U.S. Census Bureau Fact Finder); that 181 (69.6%) of the students in that ZIP code sample lived in two counties that have a higher poverty rate than the state figure (2013 poverty rates: State: 13.5%; County A: 18.4%; County B: 16.5%) (Ball, 2013); and that in 2013 the poverty rate for Hispanic households in the state was 2.5 times that for White non-Latinos (24.2% versus 9.0%) (Ball, 2013). The above information would seem to imply that the number of students who have grown up in and/or live in or near the poverty line is likely higher in developmental math classes than in the general population of this community college, given the higher percentage of Hispanics in those math classes. It is important to recognize this sub-group in the study population in light of recent reports of the adverse effects of living in poverty on the trajectory of children's brain development and learning. (Center on the Developing Child at Harvard University, 2016).

Student placement in developmental math was by standardized test (ACCUPLACER) or successful completion of a lower course (grade of C of higher). In the semester of the assessment, all on-campus sections of each course participated.

The test instrument was a line about 23 cm long, printed with the instructions on normal copy paper, with endpoints zero and twenty marked (Fig. 3). The decision to use a 0-to-20 line was based on earlier interviews with adults using Steffe and Cobb's model, where Stage of number sense could be determined with an oral missing addend question when the largest "whole" was 25 (Steinke, 1999). Also, using 20 allows those students able to do so to mentally picture the middle of the line as 10. The decision to use five numbers was based on an in-class experience with an adult student prior to developing the assessment. The given numbers were written in a vertical box and out of order – 17, 12, 2, 5, 1. The specific numbers were chosen based on: 1) avoiding 10 (a center benchmark) (Friso-van den Bos et al., 2015); 2) using only one other benchmark (either 5 or 15); 3) including 1 and 2 to show a person's sense of the "equal distance" concept (the distance from 0 to 1 and from 1 to 2 should be the same); 4) excluding numbers one more or one less than any benchmark beyond zero (thus excluding 4, 6, 9, 11, 14, 16, 19); and 5) using no consecutive numbers beyond 1 and 2 (thus excluding 3). From the remaining numbers (7, 8, 12, 13, 17, 18), two beyond 10 were chosen. This decision again was based on interviews; Stage 1 or weak Stage 2 adults began to struggle with missing addend questions in which the whole was greater than 10. The 12 and the 17 were chosen.

The assessment was usually done at the first class meeting of the semester and no later than the third class meeting. Participants were all the students present in class on the day of the assessment. After students received the assessment tool, the lead researcher or a result evaluator read the directions aloud while displaying the tool and physically pointing to the ends of the line (the zero and the twenty). If students had questions about how to proceed, a general remark such as "It's up to you." was given. Testing an entire class of up to 32 students took no more than ten minutes, including the time for distributing the assessment and reading the directions.

The tests were then analyzed separately for Stage of number sense by two different math instructors. The instructors met later to compare their separate results and arrive at a consensus on those assessments for which their original Stage placement differed. A template of the ideal (i.e., perfectly placed) location of each number was used to judge the accuracy of the responses.

Results

Stage 1 thinking appears on the assessments as positioning the five given numbers nearly equally across the number line (Fig. 4). This reflects the person's understanding that the numbers are in order but do not have a specific, physical size relationship.

It is also indicative of "must see them to count them" thinking. Numbers not listed appear to be ignored.

Stage 2 thinkers have an "either-or" understanding of the "part-whole" relationship (see Fig. 2). This causes them to focus on either the size of parts (the size of their personal, internal "1") or the size of the entire line, but not the spatial relationship of both at the same time.

Stage 2 thinking appears on the assessments as numbers that are correctly proportionally spaced unto themselves, but that are not in the correct location on the entire line. Stage 2 thinking results in two main types of errors: 1) an obvious leftward skewing of the entire set of numerals, often to the left of the center of the line (Fig. 5a) or 2) a proportional spacing of the digits 1, 2, 5, and 12 too far to the left and a proportional spacing of 17 close to 20 (Fig. 5b). In both cases, the size of "1" is internal and individual for that person. Also, because Steffe and Cobb noted that Stage 3 thinkers in the interviews arrived at the correct answer on the first try and were certain of their answers, any corrections or erasures of the original placement of a number caused the assessment to be judged Stage 2 (Fig. 5c).

Contrast Stage 2 "either – or" thinkers' assessments with those of Stage 3 thinkers who use the whole line as a reference and locate the numbers (the parts) within that distance (Fig. 6). People at Stage 3 may also mark the location of 10 and/or 15 on the line, a strong indication that they are thinking about the parts within and at the same time as the whole. Furthermore, Stage 3 thinkers have no erasures on their paper because, as Steffe and Cobb noted with Stage 3 children, they know their response is correct on the first try.

By far the majority of the assessments revealed a correct sense of number relationships on a number line. In many of these "correct" number lines, the 12 appears to be positioned slightly farther to the left than it should be. This is likely due to the well-documented Spatial-Numerical Association of Response Codes (SNARC) effect. Researchers found that humans judge the distance between two larger neighboring numbers (like 12 and 13) to be less than the distance between two smaller neighboring numbers (like 2 and 3) (Dehaene, Bossini, & Giraux, 1993; Wood et al., 2008) even though both pairs of numbers are the same-sized "1" apart.

Analysis

Recapping the parameters used in evaluating a number line for Stage placement:

Stage 1 – The five given numbers are spaced fairly equally across the line.

Stage 2 – The five given numbers are spaced somewhat proportionally to each other, but not proportionally to the entire line on the first attempt. Specific Stage 2 indicators on an assessment are: 1) the numeral 12 placed left of the midline; 2) 1, 2, and 5 skewed toward zero and 12 and 17 skewed toward 20; 3) excessive space between 17 and 20.

Stage 3 – Reasonable spacing of the five given numbers on the first attempt, allowing for the SNARC effect; no erasures; and, in some results, marking the middle of the line as a reference point.

Inter-rater Reliability

When the instructors met to compare their individual analyses, there was strong initial agreement about which students were Stage 3. In the Algebra 1 assessments, one reviewer classified 215 results as Stage 3; the other agreed with 191 of those (89%). When reaching consensus on the remaining 24 assessments, only 3 were moved higher, from Stage 2 to Stage 3. There was also strong agreement about Stage 1: of the 6 in Algebra 1, four were agreed upon immediately, and two more by consensus.

Stage 2 was more complicated because of the SNARC effect and the variations of error types (see Figure 5). How close to the exact location of the number did a student's placement have to be to qualify as Stage 3? Even so, in the Algebra 1 results, of the 59 assessments initially placed in Stage 2 by one reviewer, the second reviewer agreed with 55 of those placements, a 93% agreement rate. After discussion, a number of results were reclassified. If the two instructors could not agree on an example as Stage 2 or Stage 3, that assessment was labeled Stage 2.5. In reporting the results of this assessment set, all these uncertain-Stage results were put in the Stage 3 category. That means the final numbers reported here are very conservative.

Number Line Results

Tables 1, 2, and 3 show the percentages for the Stage of the students in each of the three courses. The first percentage is for all students who took the assessment (ALL). The second percentage includes only those students who received A, B, C, D, or F grade in the course (A to F grades) and excludes those who continued in the course after the census date but withdrew (W) prior to receiving a final grade. In fact, students who left before or after census had little to no affect on the overall percentages. Combining all three courses, 77% of those who took the assessment at the start of the term were Stage 3; 23% were not. At the end of the term, of those who had taken the assessment and received a letter grade, 78% were Stage 3 and 22% were not.

What is surprising is that there was a higher percentage of NOT Stage 3 students in Algebra 1 than in the lower-level courses. Looking at each course, the percentage of students NOT Stage 3 was 18% in Basic Math and 18% in Pre-algebra, while in Algebra 1 it was 28% . This implies that some students may be scoring high on the math placement exam even though they lack the background concept of "part-whole coexistence."

Stage of Number Sense and Math Course Success

Further analysis revealed that there is a difference in success rate in these math courses between those who have the part-whole concept (Stage 3) and those who do not. Success is defined as a final grade of A, B, or C. Including only those students who received grades of A through F, by a two-proportion z –test, the difference in success rate is significant in Pre-algebra at $p < .1$ ($p = .085$) and in Algebra 1 at $p < .05$ ($p = .039$). The difference was not statistically significant in Basic Math.

Furthermore, letter grades in all three courses for students who passed are skewed toward A and B for Stage 3 students and toward C for Stage 1 and 2 students as shown in Figures 7, 8, and 9. Note also that the percent of students who withdrew from each course after the census date but without receiving a grade (W) was higher for Stage 1 or 2 students than for Stage 3.

It is true that students may withdraw for job-related, family-related, or health-related reasons throughout the semester. However, anecdotal evidence, including from instructor gradebooks, indicated that students who withdraw just before the deadline (the end of week 13 of a 15-week semester) are more likely not to be passing the course at that point. Withdrawing avoids a poor grade. (Note that students who withdrew (W) are included in Figures 7, 8 and 9, making that the total number of students different from that in the Tables.)

"Rules" for Analysis of Future Tests

After the original "eyeball" analysis of the test results, a more rigorous analysis of the physical data was undertaken. Each marked point on each number line was measured by hand to the nearest .5 millimeter. When erasures were detected on the page, the original point(s) were measured as the person's response. The difference of each point was computed plus-or-minus from the exact ideal location of that point on the number line. The ratio of the distances between each two neighboring points was also computed. The math instructors' visual classification of results was then compared with these numbers to attempt to find some general rules for reducing subjectivity in future number line assessment classification.

Stage 1 students' results generally were found to have ratios of the distance between neighboring numbers that approached 1 in at least three of the four comparisons where the ratio should not have been 1. This is the "equal spacing" that was noted in the visual classifying.

To attempt to find a rule for Stage 3, the Stage 3 assessments for Basic Math (137) and Pre-Algebra (146) were used. The mean of each of the five points from those results was taken as a benchmark and simple Standard Deviations (SD) from those benchmarks were computed. These parameters were then applied to the 311 Algebra 1 assessments.

It appeared that a criterion of all five points of the assessment falling within 1.5 SD from the benchmarks (that first set of Stage 3 means) might be a good sorting mechanism for Stage 3. In the Algebra 1 data, 184 of the 224 assessments identified by visual inspection and consensus as Stage 3 (165) or Stage 2.5 (19) (those uncertain results that were bumped to the higher level) meet the 1.5 SD criterion. That is, this 1.5 SD criterion sort matches 82% of the visual inspection sort.

These numerical results seem to support the trained math instructors' visual classification as being adequate as a quick first look for students at Stage 1 and Stage 3.

Stage 2 had no general numerical rule that could be deduced from the Stage 3 Standard Deviation data. This may be in part because of the variety of errors on Stage 2 number lines. Also, only 81 assessments from Algebra 1 were classified as Stage 2 by visual inspection. That did not provide enough examples of each type of error to arrive at measurement-based rules for Stage 2 beyond "12 placed left of center." In the Algebra 1 course, 23 of the 81 Stage 2 results (28%) met this criterion.

A much larger set of assessments would need to be gathered to determine whether these criteria apply to the general population. Using newer technology (such as a pen that writes on a tablet or computer surface) and the GeoGebra software program (which can measure the distance between two points on a line automatically) a large-scale test would seem to be feasible.

Significance

The concept of part-whole coexistence is critical for understanding proportions, fractions and algebraic relationships. The concept is also central to the College and Career Readiness Standards (CCRS) (Pimental, 2013) around which the new adult high school equivalency tests are built. Students' positioning of non-sequential whole numbers on the empty line appears to reveal whether they grasp that concept and have arrived at mature number sense.

The results of this study suggest that over 20% of developmental math students in this sample have not. This is in line with results from the 2003 National Assessment of Adult Literacy (NAAL) (which included numeracy) showing 22% of adults in the United States at below-basic level in math (U.S. Department of Education, 2011). The recent PIAAC international test of adult numeracy (U.S. Department of Education: PIAAC, 2014) indicated similar math deficiencies: 30% of American adults were below or at Level 1, compared to the international average of 19%.

Remediation

How can this picture be changed? Adult students are apt to resist revisiting primary-grade-level concepts (see Figs. 1 & 2) if instruction is undertaken in a purely mathematical context.

Effecting conceptual change is more likely to be successful when new ideas are linked to students' personal experiences. Below are brief descriptions of some of the ways this instructor has addressed key concepts, including the meaning of the equals sign. Changing students' understanding of that symbol from "operation" to "relationship" (Wheeler, 2010; Knuth et al., 2008) is required prior to addressing the equivalency relationship implicit in the part-whole coexistence and "equal distance" concepts.

1) Equals sign: Use the full name and nickname of several students. On the board, write an equals sign between each set of names, stressing "different name, same person." Follow up with examples of equivalent expressions with different operations, such as 17 – 9 = 4 x 2 and "different name, same amount."

2) Part-whole coexistence: Have students name the parts of an object (a chair, a car). Ask if the object is complete if a part is missing. Ask if the parts continue to exist within the object when speaking of the whole object. Follow up with missing addend and missing factor word problems with misleading "key words." Encourage students to think of the number information in the problems in terms of the part-whole coexistence relationship.

3) Equal distance between whole numbers: Ask students to trace with a finger the <u>spaces</u> between the marks of a 1-unit number line *at a steady beat* (Fig. 10). Use a digital metronome (marking equal spaces of time) set at the students' comfortable body speed. Be sure students place their tracing finger on the zero mark to start. Follow up with lessons on line graphs or the coordinate grid, emphasizing the <u>equal spaces</u> <u>between</u> the lines, not the digits.

Suggestions for Further Research

The revelation of the degree to which the two concepts, equal distance and part-whole coexistence, are lacking in adult students makes this area ripe for further investigation. The utility and reliability of this number line assessment could be compared to that of standard computation-based math placement exams when determining a student's appropriate starting point for math remediation and/or course placement. Another interesting avenue would be to compare number line assessment results with tests of critical thinking skills or reading comprehension, both of which also require considering the parts and the whole at the same time.

The topic of remediation for students lacking the concepts is also open for research. What tools and materials are most effective? Will whole-class instruction work? Does remediation with adults need to be one-on-one?

Implications for the Field

Current mainstream adult basic education math texts and college developmental math texts do not explicitly teach either of the missing concepts, "equal distance" and "part-whole coexistence" with <u>whole numbers</u>, and that concept's necessary precursor, the equals sign as relationship. It would seem the texts assume that adults grasp these concepts. Such an assumption may exist in math curricula as early as fourth grade, about age 9. That is the age at which the brains of students living in the toxic stress of poverty (Child Welfare Information Gateway, 2015) are perhaps just beginning to grow the connections that allow the student to keep two things in mind at the same time, a pre-requisite for understanding part-whole <u>coexistence</u>. This brain growth often happens for children living in more secure environments at about age 8 (Rueda et al., 2004), which is 3rd grade, and seems to be secure for 9-year-olds (Poirel et al., 2012), which is 4th grade.

As noted earlier, many of these developmental math students likely come from low socio-economic backgrounds, where toxic stress delays "normal" brain development. Other students may have been the youngest in their class (or nearly so), so their brain development was later than their classmates, the "relative age effect" documented by Bedard and Duhey (2006). Whatever the cause, the Stage 1 and Stage 2 adult students were not able to grasp the concepts when they were presented in the primary grades. Until the brain development is there, teaching these two concepts is like expecting a color-blind person to be able to learn to distinguish between lime green and chartreuse.

The ultimate solution would seem to lie in aligning the elementary math curriculum with students' neurological development rather than chronological age. The system needs to wait until the brain is ready before presenting abstract concepts that require part-whole thinking. In the meantime, the quick assessment presented here may be a useful tool for teachers to determine the true root of many adults' difficulty with part-whole relationships in fractions and decimals, and to lead to appropriate explicit instruction in those concepts for those adults. Such instruction will meet the needs of more students and allow them to be more successful in math. ❖

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Table 1— *Percent of BASIC MATH students at Stage 3, Stage 2, and Stage 1*

Based on the Number Line Assessment of Number Sense				
Basic Math	**ALL**		**A to F grades**	
NUMBER	179		160	
Stage 3	146	81.6%	131	81.9%
Stage 2	17	9.5%	15	9.4%
Stage 1	16	8.9%	14	8.8%

Table 2— *Percent of PRE-ALGEBRA students at Stage 3, Stage 2, and Stage 1*

Based on the Number Line Assessment of Number Sense				
Pre-Alg.	**ALL**		**A to F grades**	
Number	167		137	
Stage 3	137	82.0%	114	83.2%
Stage 2	18	10.8%	14	10.2%
Stage 1	12	7.2%	9	6.6%

Table 3—*Percent of ALGEBRA 1 students at Stage 3, Stage 2, and Stage 1*

Based on the Number Line Assessment of Number Sense				
Algebra 1	**ALL**		**A to F grades**	
NUMBER	311		247	
Stage 3	224	72.0%	182	73.7%
Stage 2	81	26.1%	61	24.7%
Stage 1	6	1.9%	4	1.6%

For Tables 1, 2, and 3:
ALL includes students who dropped before census or withdrew with no grade after census.
A to F includes only those tested who also received a letter grade.

Figure 1—*Stage 1 versus Stage 2 understanding of number relationships*

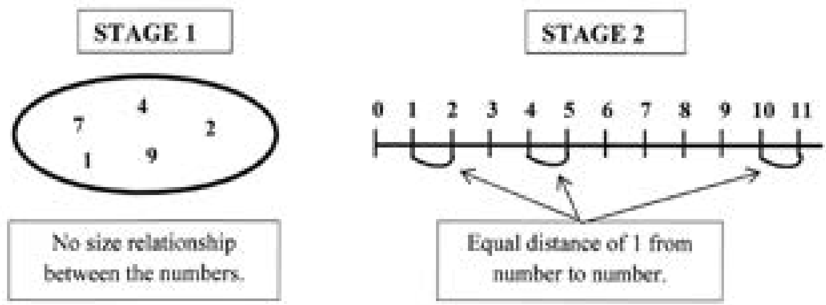


Figure 2—*Stage 2 versus Stage 3 understanding of number relationships*

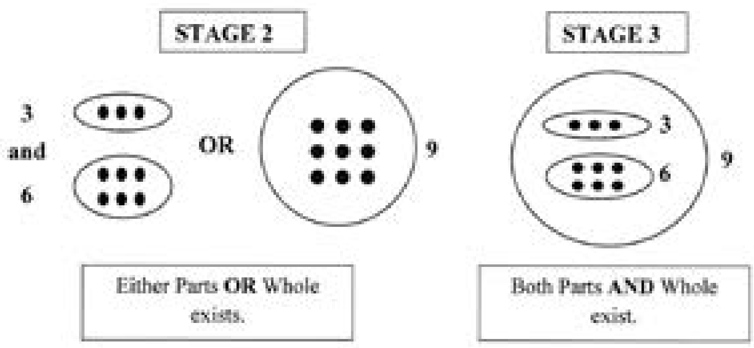
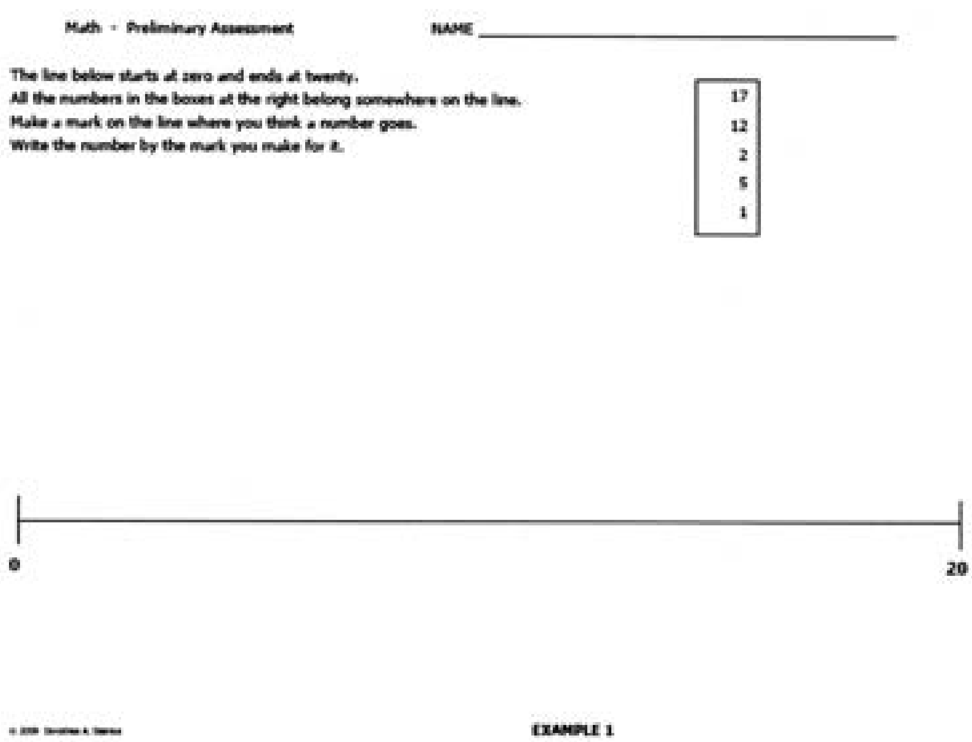


Figure 3—*Assessment Tool*



Math - Preliminary Assessment NAME ____________________

The line below starts at zero and ends at twenty.
All the numbers in the boxes at the right belong somewhere on the line.
Make a mark on the line where you think a number goes.
Write the number by the mark you make for it.

17
12
2
5
1

0 20

EXAMPLE 1

Figure 4—*Stage 1 Number Line*

All numbers nearly equally spaced across the line.

Figure 5—*Stage 2 Number lines*

a) Numbers skewed left

b) Numbers skewed toward ends; 12 left of center

c) Excess space between 5 and 12; correction of placement

Figure 6—*Stage 3 Number Line*

Correct relationship of parts within the whole line.

Figure 7—*Basic Math Grade Distributions*

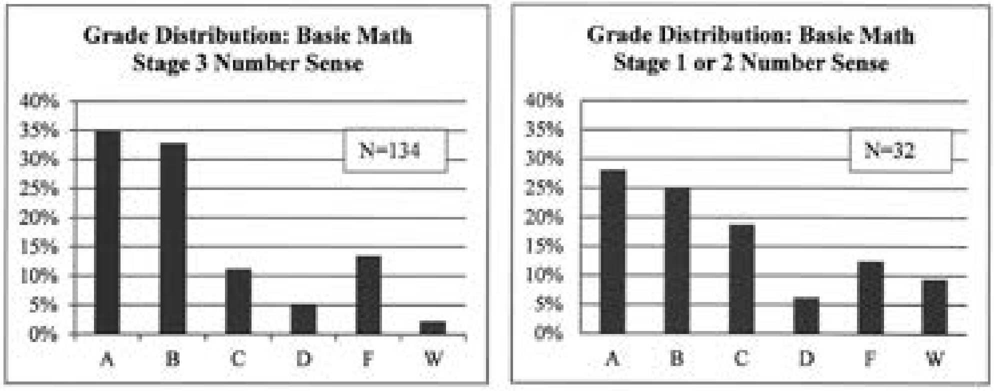


Figure 8—*Pre-algebra Grade Distributions*

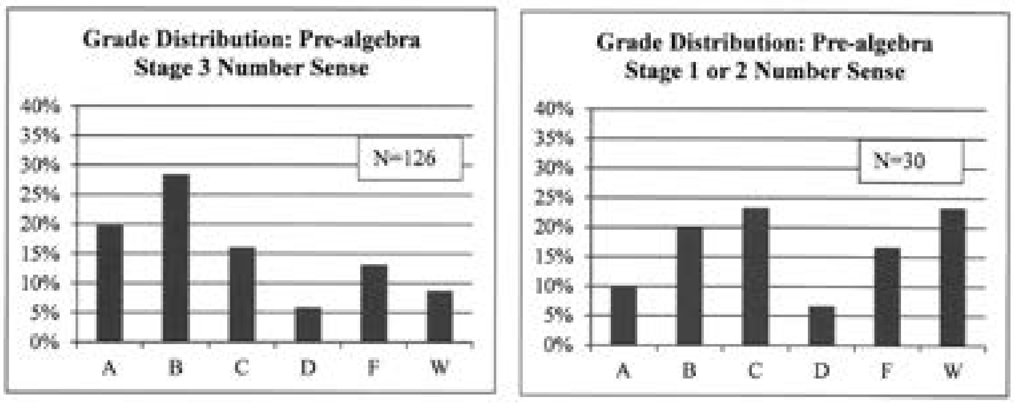


Figure 9—*Algebra 1 Grade Distributions*

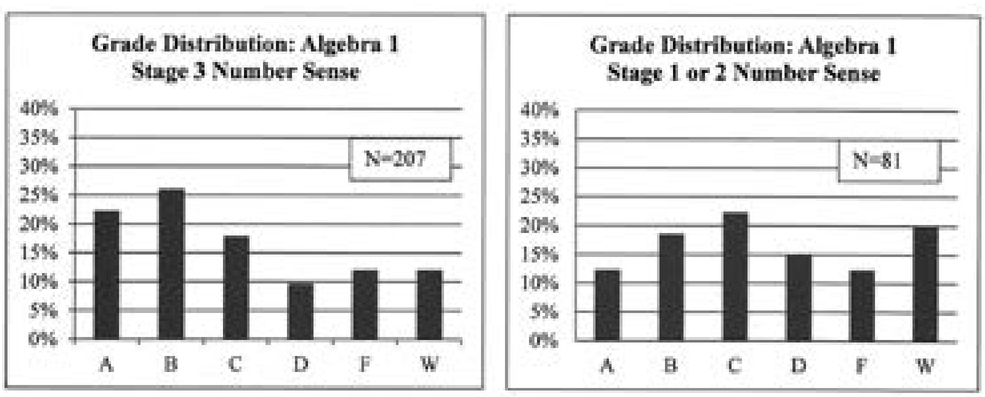


Figure 10—*Sample of number line with guide to trace spaces*